GNSS Standard Positioning Service

An introduction to the
GNSS Standard Positioning
Service

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C/A-codes and modulation

To make sense of the Standard Positioning Service (SPS), it is important to understand the C/A-code concept and how it is used to transmit the information required for ranging, and thus determining a position.

Like any other radio message, GNSS signals are transmitted by the satellites at certain frequencies. The best known of these is called L1 which is a frequency of 1575.42 Mhz. It is also referred to as a carrier wave frequency - "carrier" because it is used to carry other information to a receiver.

A carrier wave can be used to carry information by modulating its frequency - modulation gives the carrier wave a shape that represents the information. The shape is understood by the receiver and the information recovered.

Every GNSS satellite is allocated a unique Pseudo Random Noise (PRN) code (C/A-code) which is modulated onto the satellite's carrier wave frequency.

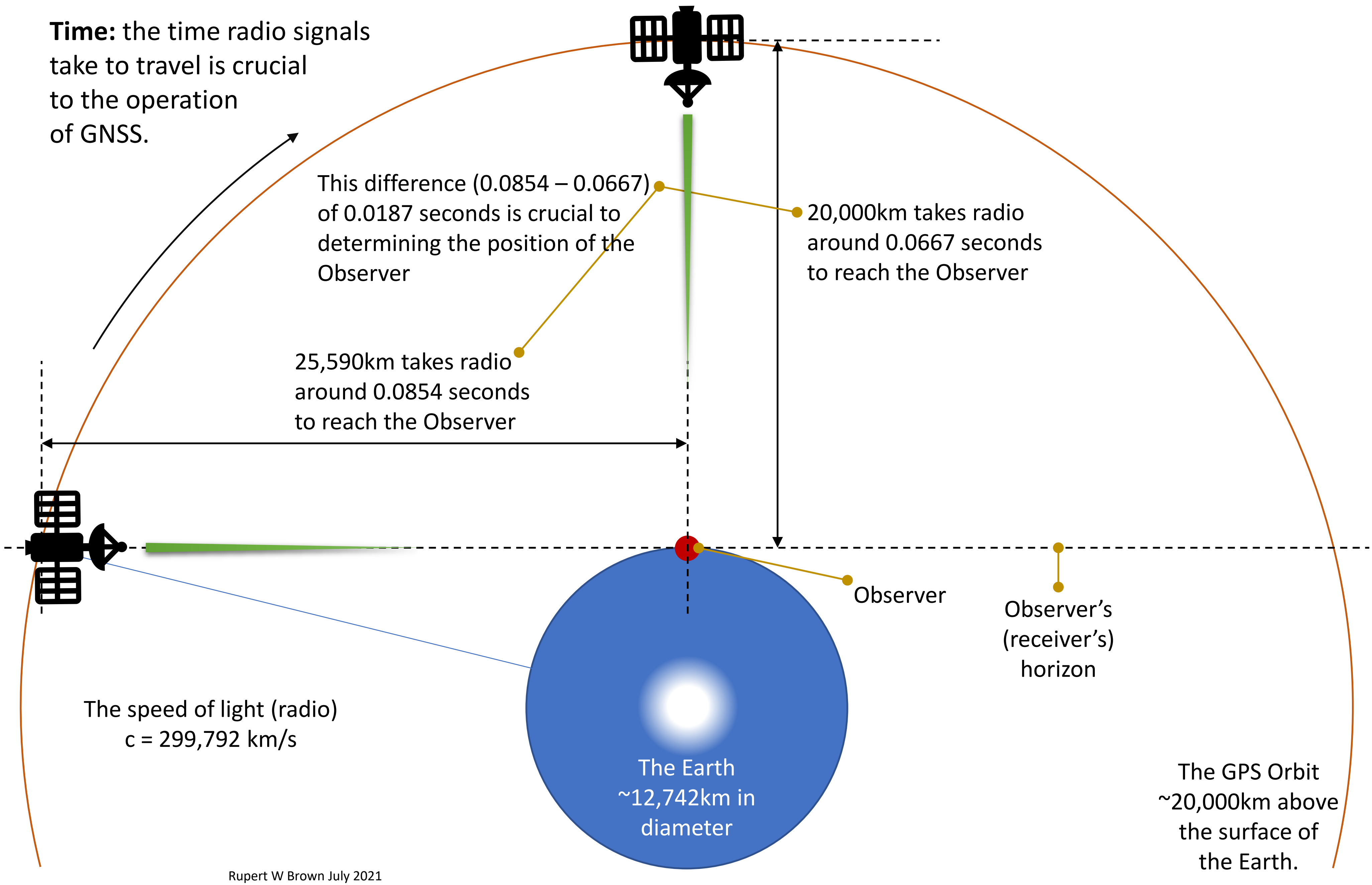
The position of the satellite in its orbit around the Earth is contained within a navigation message. The data in this message is overlaid on the C/A-code. A GNSS receiver can identify the satellite through the PRN code and decode the navigation message. Timestamps informing the receiver when the satellite sent the message, are embedded as part of the modulation process.

Thus the receiver can determine where the satellite is, and how far it is from the satellite. If a receiver can see four or more satellites and has at least four sets of satellite positions and distances, it can calculate its position.

Finding a position using GNSS

The basic process stages for the Standard Positioning Service (SPS) are:

- The systems on the ground - the monitoring and ground control stations - maintain a very close watch on the GNSS satellites in orbit. Central processing systems calculate very precise satellite orbits and adjustments to satellite clocks which are broadcast to the satellites as they pass over ground control stations.
- The satellites use the data from the ground control stations and create their GNSS signal. They modulate the carrier wave with their PRN (C/A-code) and the navigation message, and broadcast to Earth on a continuous basis.
- A receiver picks up the GNSS signals. A receiver has a number of channels. Each channel can process the data from one satellite.
- The receiver decodes the GNSS signals. The receiver identifies the satellite by decoding its PRN code and matches it to a list of known satellites. It determines the position of that satellite, and how far it is from the satellite, from the navigation message and timestamp information.
- Algorithms are used to turn these satellite positions and distances into a series of X, Y and Z coordinates.
- Trigonometry, specifically a technique called trilateration, turns the X, Y and Z coordinates and distances of several satellites into the X, Y and Z position of the receiver.
- Finally the receiver can translate its own X, Y and Z coordinates into latitude, longitude and height (using reference frames).


Time: the time radio signals take to travel is crucial to the operation of GNSS.
This difference (0.0854 – 0.0667) of 0.0187 seconds is crucial to determining the position of the Observer
20,000km takes radio around 0.0667 seconds to reach the Observer
25,590km takes radio around 0.0854 seconds to reach the Observer
Observer
Observer's (receiver's) horizon
The speed of light (radio) c = 299,792 km/s
The Earth ~12,742km in diameter
The GPS Orbit ~20,000km above the surface of the Earth.
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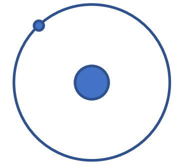
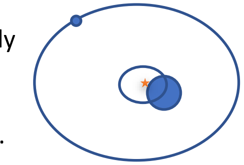
Location in orbit: specifying the location of a satellite – the navigation message

This diagram represents the orbit as circular with an *eccentricity* of 0.

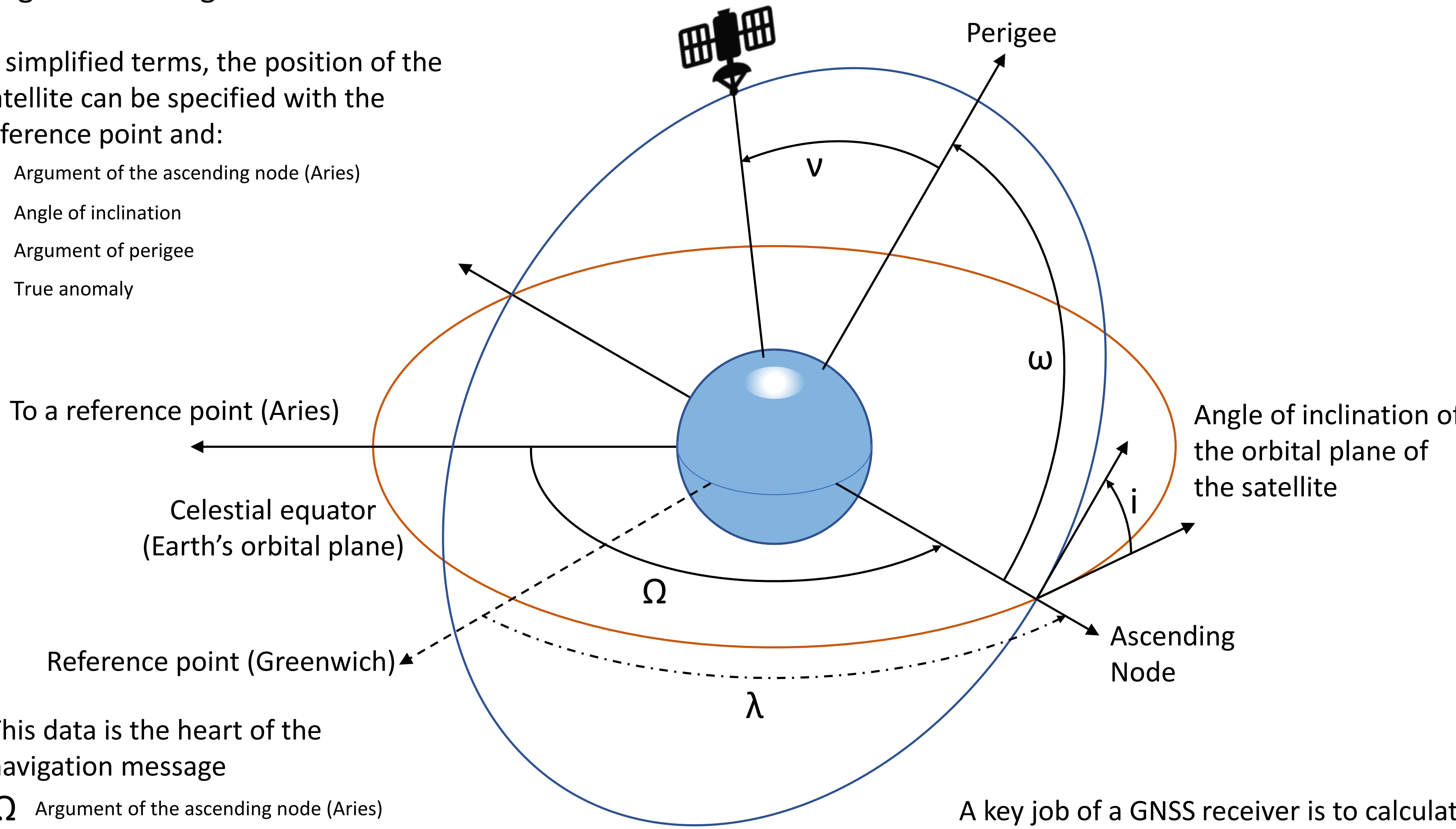


The reality is that two bodies usually have elliptical orbits around their centre of mass – barycentre – and have an eccentricity greater than 0.



In simplified terms, the position of the Satellite can be specified with the reference point and:

- Ω Argument of the ascending node (Aries)
- i Angle of inclination
- ω Argument of perigee
- ν True anomaly



This data is the heart of the navigation message

- Ω Argument of the ascending node (Aries)
- i Angle of inclination
- ω Argument of perigee
- ν True anomaly

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A key job of a GNSS receiver is to calculate the position of a satellite using Ω i ω and ν, then convert that position into the X, Y and Z of the Terrestrial Reference Frame.

The form of the signal from GNSS satellites

The navigation message and the PRN code are “mixed together” with the carrier frequency to create a composite broadcast signal.

GNSS signals are transmitted across several frequencies. For GPS these are L1 (1575.42 Mhz), L2 (1227.60 MHz) and L5 (1176.45 MHz).

The GNSS signal carries navigation data:

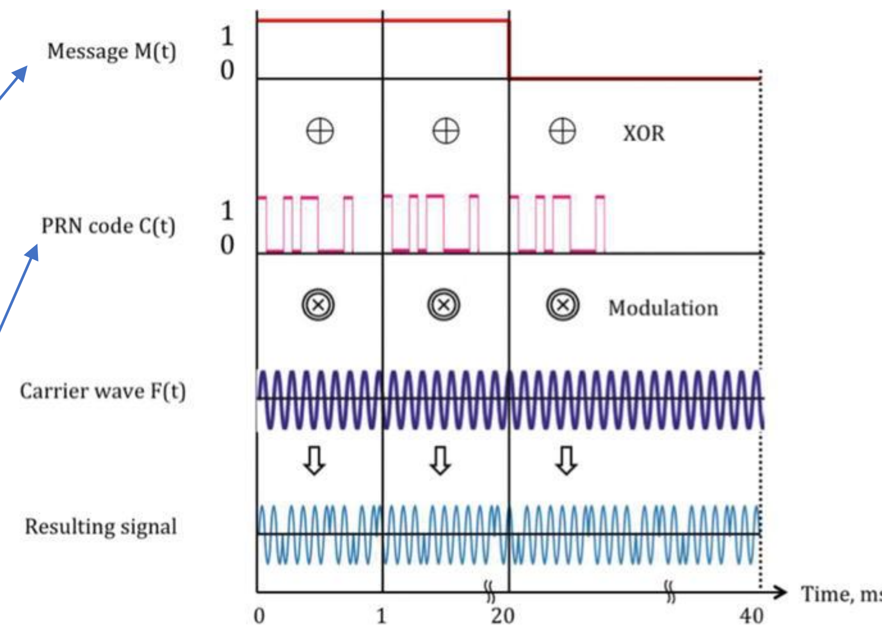
Parameter	Explanation
t_{oe}	Ephemerides reference epoch in seconds within the week
$\sqrt{a}$	Square root of semi-major axis
e	Eccentricity
M_o	Mean anomaly at reference epoch
ω	Argument of perigee
i_o	Inclination at reference epoch
Ω_0	Longitude of ascending node at the beginning of the week
Δn	Mean motion difference
$\dot{i}$	Rate of inclination angle
$\dot{\Omega}$	Rate of node's right ascension
C_{uc}, C_{us}	Latitude argument correction
C_{rc}, C_{rs}	Orbital radius correction
C_{ic}, C_{is}	Inclination correction
a_0	Satellite clock offset
a_1	Satellite clock drift
a_2	Satellite clock drift rate

This data is organised into 25 frames. Each frame is Divided into five subframes containing 10 30 bit words. The whole message takes 12.5 minutes to transmit at 50 bps. The navigation message modulates the carrier frequency.

The ranging signal is based on a pseudo random number (PRN) code. Each satellite has a unique PRN code of 1,023 bits which is repeated every millisecond.

The PRN code is used to modulate the signal frequency (carrier wave).

To prepare the GNSS signal for transmission by the satellite, first an XOR operation is applied to combine the binary navigation message with the code. If the message bit and the code chip are the same, the result is 0; if they are different, the result is 1. Second, the combined signal is merged with the carrier using binary phase shift keying (BPSK) modulation: a “0” bit leaves the carrier signal intact, whereas a “1” bit causes the signal to be multiplied by −1 and shifts the carrier by 180°.

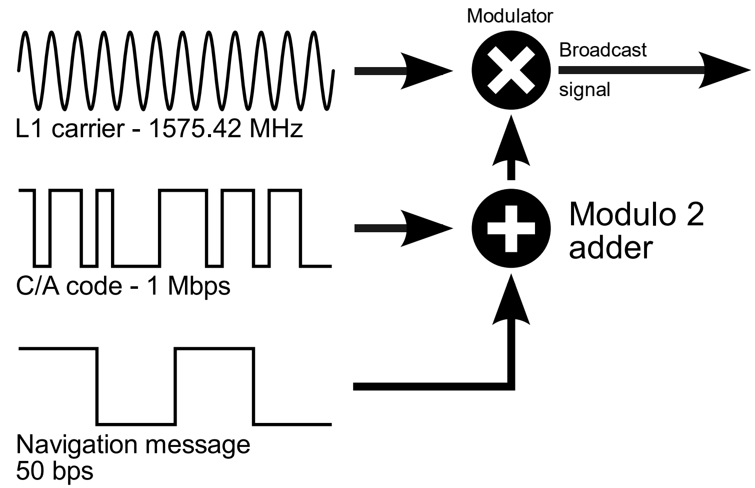


Ref: “GNSSs, Signals, and Receivers” By Mohamed Tamazin, Malek Karaim and Aboelmagd Noureldin
“GNSS DATA PROCESSING Volume I: Fundamentals and Algorithms” by J. Sanz Subirana, J.M. Juan Zornoza and M. Hernández-Pajares

Who is who?: CDMA, chips and codes

If all the GNSS satellites are broadcasting on the same frequency, how come the signals don't get mixed up? How do we know what comes from where?

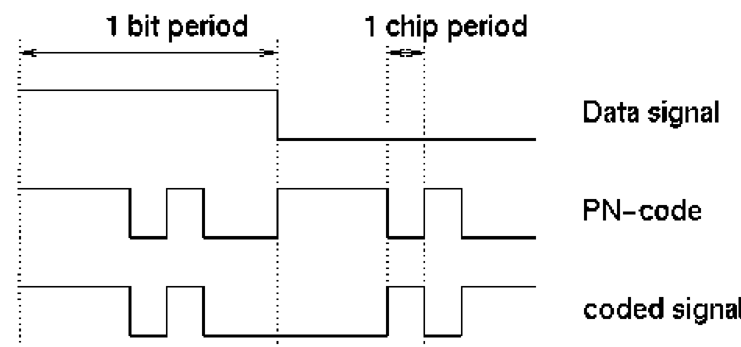
The answer to this puzzle lies in understanding how Code Division Multiple Access (CDMA) works. Every GNSS satellite is allocated the entire transmission band all of the time. The trick is that each satellite's message is coded using their unique PRN code. The key to CDMA is to be able to extract the desired signal while rejecting everything else as random noise.



By P. F. Lammertsma, converted to vector by Denelson83 - Satellite Navigation, P. F. Lammertsma, p. 9, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=1383669

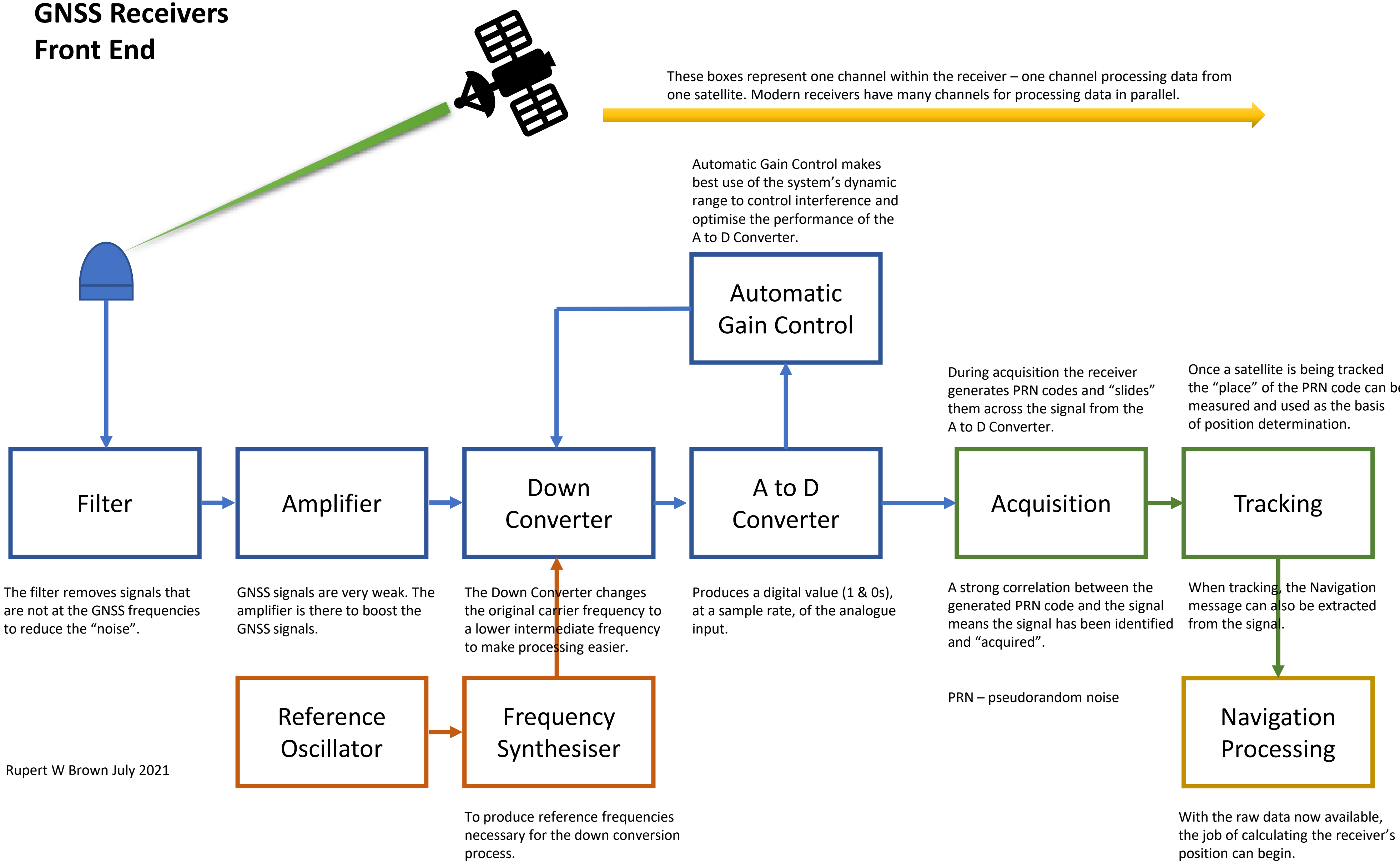
In CDMA, each message bit is subdivided into m short intervals called chips. Each satellite is assigned a unique 1,023 bit chip sequence (the PRN code). To transmit 1 bit of the navigation message, a satellite sends its chip sequence. To transmit a 0 bit, it sends the one's complement of its chip sequence. For example, if satellite A is assigned the chip sequence 00011011, it sends a 1 bit by sending 00011011 and a 0 bit by sending 11100100.

For GPS, the navigation message is sent at the very sedate rate of 50 bits per second. The rate at which the chips are sent (the chipping rate) is 1.023 mega bits per second. This means that a chip (PRN code) is transmitted in a microsecond. The navigation message and chips modulate the carrier frequency – for L1 this is 1575.42 Mhz.



The wave length of the carrier frequency – for L1 this is 1575.42 Mhz – is c = 299,792,000 m/s divided by 1,575,420,000 hz which gives 0.1903 m or about 19 cm. This is significant for *carrier phase* positioning.

GNSS Receivers Front End



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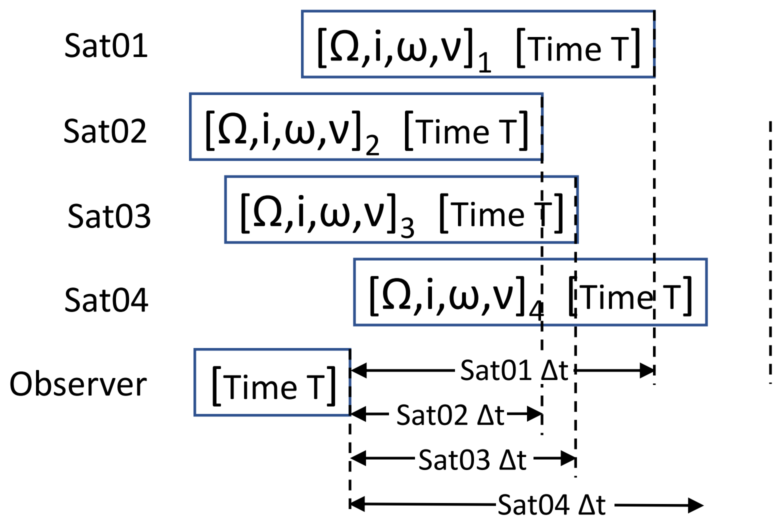
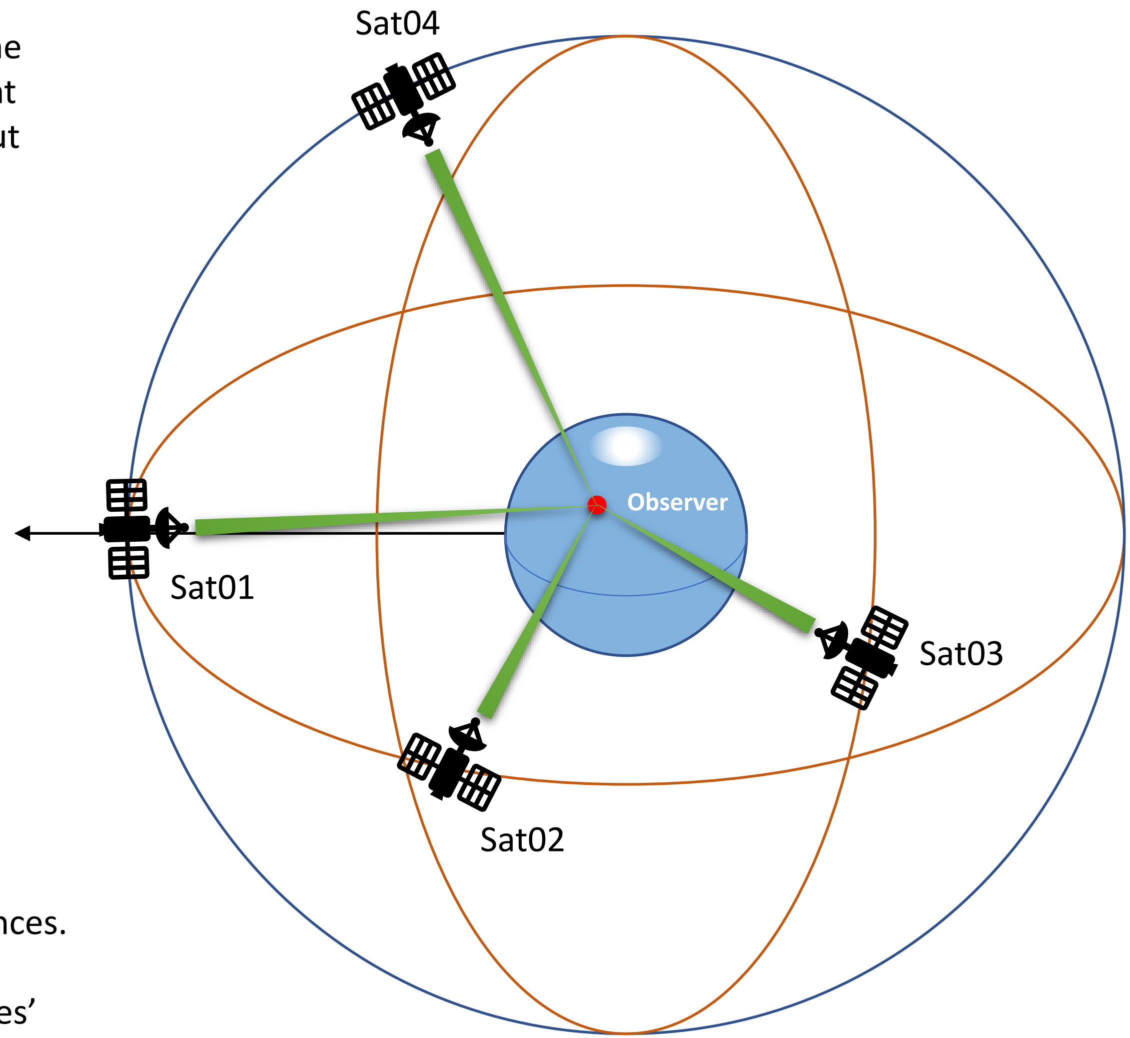
Position: determining a position on Earth using GNSS radio signals

Navigation messages and PRN codes sent from the satellites are received by the Observer at different times. The receiver uses the PRN code to work out the distance to the satellite.





Messages sent at the same time by the satellites are received at different times by the Observer because the signals have to travel different distances. These time differences can be used to calculate those distances. With a knowledge of the satellites' positions and their distance from the Observer, the Observer can calculate its position on the Earth's surface.

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Navigation Processing
The Navigation Message

The basic process is:

Compute the time t_k from the ephemerides reference epoch t_{oe} (expressed in seconds in the GPS week)

Compute the mean anomaly for t_k

Solve (iteratively) the Kepler equation for the eccentric anomaly E_k

Compute the true anomaly v_k

Compute the argument of latitude u_k from the argument of perigee ω, true anomaly v_k and corrections c_{uc} and c_{us}

Compute the radial distance r_k, considering corrections c_{rc} and c_{rs}

Compute the inclination i_k of the orbital plane from the inclination i_o at reference time t_{oe}, and corrections c_{ic} and c_{is}

Compute the longitude of the ascending node λ_k (with respect to Greenwich)

Compute the coordinates in the TRS frame, applying three rotations (around u_k, i_k and λ_k) => X, Y and Z

(X_n, Y_n, Z_n)

“GNSS DATA PROCESSING Volume I: Fundamentals and Algorithms” by J. Sanz Subirana, J.M. Juan Zornoza and M. Hernández-Pajares

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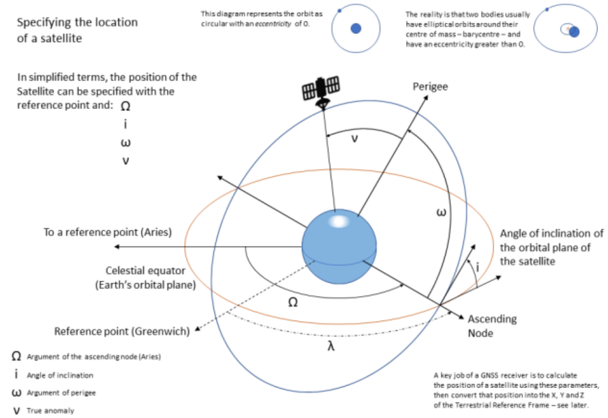
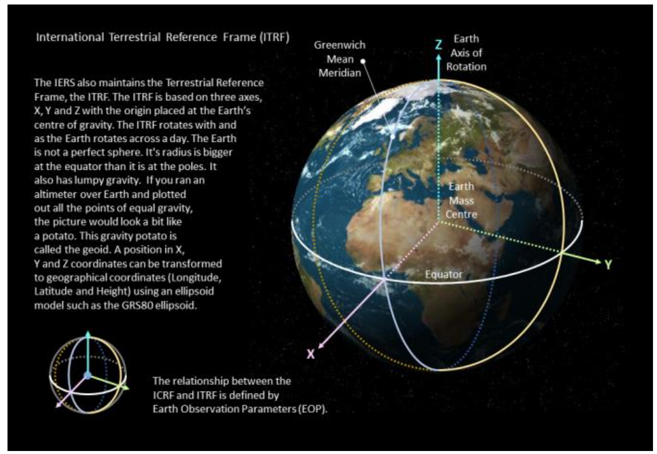
The data in that Navigation message



The meaning of those terms



The X, Y, Z reference frame

Navigation Processing
The Distance Message (Pseudorange)

Pseudorange = **Speed of Light** × [**Receiver Clock Time** − **Satellite Clock Time**]

For a given signal frequency

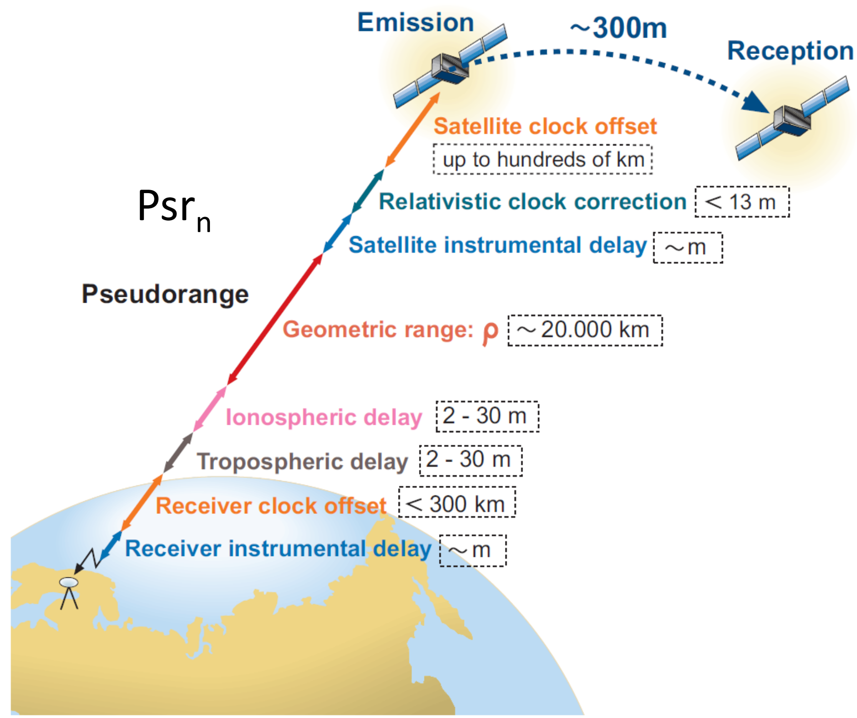
\+ An adjustment to take into account the physical phase centre of the antennas in relation to the the point whose position is being measured.

\+ The delay due to the Troposphere.

\+ The delay due to the Ionosphere. (For a given signal frequency)

\+ The delay caused by the electronics within the satellite and receiver – biases. (For a given signal frequency)

\+ Signal noise in the receiver.



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Navigation Processing
Putting it together – the position

Start with a point corresponding to the approximate position of a receiver.

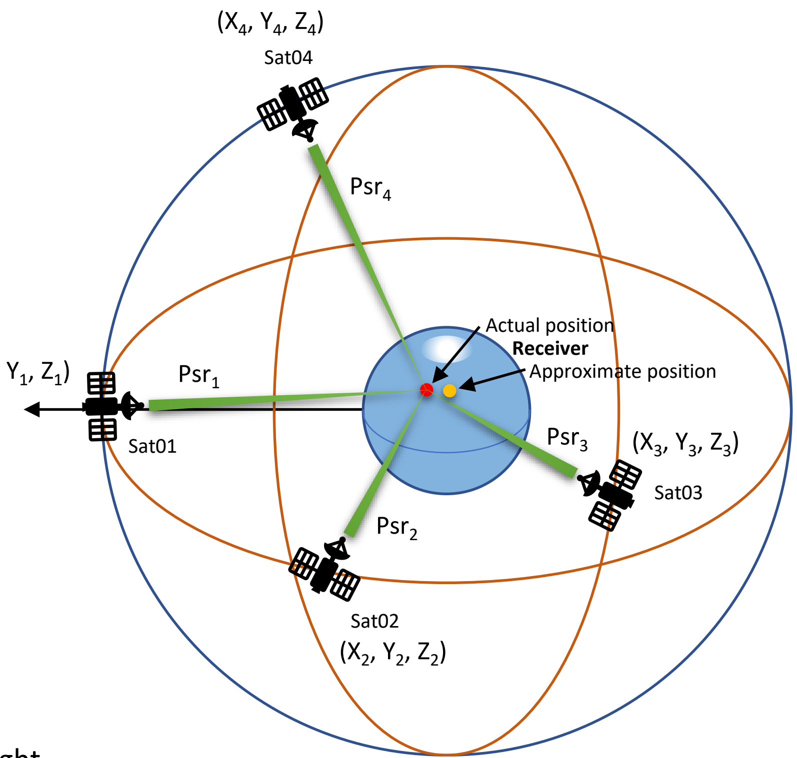
Creating a n by n matrix, with n = number of satellites, containing:

X, Y, Z delta to the approximate position:
Based on that approximate position, the position of the satellite and the pseudorange, derive a linear approximation (linearization) to the actual position – as an X, Y, Z delta to the approximate position.

Find the actual position *best fit* by solving the matrix equation using a least squares adjustment and producing a new value for the approximate position of the receiver.

Iterate again with this new value of the approximate position of the receiver till the X, Y, Z deltas to the approximate position fall below an acceptable threshold – **the receiver's position is found**.

Convert X, Y and Z into latitude, longitude, height.



This can be considered to be a Standard Positioning Service (SPS).

The Accuracy of SPS

A report produced by the US William J. Hughes Technical Center for the US Federal Aviation Administration in 2017 states that the Standard Position Service gives a position that is:

- Within 3.9 m of the actual vertical position 95% of the time, and
- Within 1.9 m of the actual horizontal position 95% of the time.

That sounds pretty good. However, the actual performance you experience depends on many things including, but not limited to:

- The quality of your receiver. The quality of your GNSS antenna and the electronics behind it can have significant impacts on performance,
- The number of satellites your receiver can see. Generally the more satellites, the more data, the better the position,
- Multi-path effects. If the satellite signals are bouncing around nearby structures, the reflected signals can confuse the receiver,
- The accuracy of the navigation message transmitted by the satellite. If, for whatever reason, the satellite clock is a little bit off, or the satellite is a little out of its orbit then the calculated position can be thrown out too.
- The atmosphere. Both the ionosphere and troposphere can have effects on the signals which can add errors to a position.

The other signals and codes broadcast by GNSS satellites can improve that accuracy. Now there are also augmentation systems available which send additional signals that greatly improve accuracy and often carry signal integrity information. Some of these are available to the general public and some are not.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